

# A Level Summer Work

# **Answers**

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# **Laws of Indices**

### Things to remember:

$$a^{m} \times a^{n} = a^{m+n}$$

$$a^{m} \div a^{n} = a^{m-n}$$

$$a^{m} \div a^{n} = a^{m-n}$$

$$a^{0} = 1$$

$$a^{m} = \sqrt[n]{a^{m}}$$

$$a^{m} = \sqrt[n]{a^{m}}$$

### **Questions:**

(b)

- 1. (a) Simplify  $m^5 \div m^3$ 
  - Simplify  $5x^4y^3 \times x^2y$   $5x^6y^4 \dots$
  - (2) (Total for Question is 3 marks)
- 2. Write these numbers in order of size. Start with the smallest number.
  - 5-1 0.5 -5 50
  - .....-5,5<sup>-1</sup>, 0.5,  $5^0$

(Total for Question is 2 marks)

.....m<sup>2</sup>.....

3. Write down the value of  $125^{\frac{2}{3}}$ 

(Total for question is 1 mark)

25

**4.** (a) Write down the value of  $10^{-1}$ 

.....0.1....**(1)** 

(b) Find the value of  $27^{\frac{2}{3}}$ 

.....9....**(2)** 

(Total for Question is 3 marks)

5.	(a)	Find the value of	5°	1
	(b)	Find the value of	$27^{1/_{3}}$	(1)
	(c)	Find the value of	2 <sup>-3</sup>	3(1)
				1/8 <b>(1)</b>
				(Total for Question is 3 marks)
6.	(a)	Write down the valu	ue of $27^{1/3}$	3
	(b)	Find the value of 2	$7^{-1/2}$	(1)
				$\frac{1}{3\sqrt{3}}$
				(2) (Total for Question is 3 marks)
7.	(a)	Write down the valu	ue of $64^{\frac{1}{2}}$	
				8
	(b)	Find the value of $\left(\frac{1}{1}\right)$	$\frac{8}{25}$ ) $^{-\frac{2}{3}}$	
				05/4
				25/4 <b>(2)</b>
8.	(a)	Write down the valu	ue of 6 <sup>0</sup>	(Total for question = 3 marks)
				1
(b)	Work o	ut 64 <sup>-2/3</sup>		
				1/16 <b>(2)</b>
				(Total for question = 3 marks)

# **Surds**

### Things to remember:

- √ means square root;
- · To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.

### **Questions:**

1. Work out

$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

$$= \frac{22}{\sqrt{22}} = \frac{22\sqrt{22}}{22} = \sqrt{22}$$

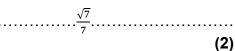
$\dots \sqrt{2}2\dots$	
	(Total 3 marks)

2. (a) Rationalise the denominator of  $\frac{1}{\sqrt{3}}$ 

$$\frac{\sqrt{3}}{3}$$

(1)

(b) Expand  $(2 + \sqrt{3})(1 + \sqrt{3})$ Give your answer in the form  $a + b\sqrt{3}$  where **a** and **b** are integers. 3. (a) Rationalise the denominator of  $\frac{1}{\sqrt{7}}$ 



(b) (i) Expand and simplify  $(\sqrt{3} + \sqrt{15})^2$  Give your answer in the form  $a + b\sqrt{3}$  where a and b are integers.

......18 + 
$$6\sqrt{5}$$
......

(ii) All measurements on the triangle are in centimetres.ABC is a right-angled triangle.k is a positive integer.

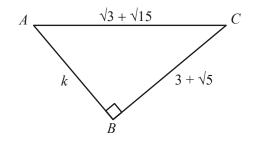


Diagram **NOT** accurately drawn

Find the value of *k*.

$$k^{2} + (3 + \sqrt{5})^{2} = (\sqrt{3} + \sqrt{15})^{2}$$
$$k^{2} + 14 + 6\sqrt{5} = 18 + 6\sqrt{5}$$
$$k^{2} = 4$$

						5-2√6		(Total 2 marks
5. (	a)	Write down the	value of 49 <sup>1/2</sup>	2		7	,	(1
(	b)	Write $\sqrt{45}$ in th	e form $k\sqrt{5}$ , w	/here <i>k</i> is an i	nteger.			(1
						3	√5	(1 (Total 2 marks
<b>5</b> . V	Write <sup>-</sup>	$\frac{\sqrt{18} + 10}{\sqrt{2}}$ in th				integers.		(Total 2 marks
				$\frac{3\sqrt{2}+10}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$	2 <del>2</del> 2			
						a =		
						b =		(Total 2 marks

Expand and simplify  $(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$ 

4.

7. Expand and simplify  $(2 + \sqrt{3})(7 - \sqrt{3})$ Give your answer in the form  $a + b\sqrt{3}$  where a and b are integers.

 $11+7\sqrt{3}-5\sqrt{3}$ 



8. Rationalise the denominator of  $\frac{(4+\sqrt{2})(4-\sqrt{2})}{\sqrt{7}}$  Give your answer in its simplest form.

9. Show that  $\frac{(4-\sqrt{3})(4+\sqrt{3})}{\sqrt{13}}$  simplifies to  $\sqrt{13}$ 

$$=\frac{16 - 4\sqrt{3} + 4\sqrt{3} - 3}{\sqrt{13}}$$

$$=\frac{13}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{13\sqrt{13}}{13} = \sqrt{13}$$

(Total for question = 2 marks)

# **Expanding and Factorising (Single Brackets)**

# Things to remember:

- Expand brackets means to multiply what is outside the bracket with everything inside the bracket.
- Factorising is the opposite of expanding put the HCF outside the brackets to factorise fully.

<b>Questions:</b>	Q	u	е	S	ti	0	n	S	:
-------------------	---	---	---	---	----	---	---	---	---

1.	(a)	Expand $5(m+2)$	
			5m+10
	(b)	Factorise $y^2 + 3y$	(1)
			y(y+3)
	(0)	Simplify $a^5 \times a^4$	(1)
	(c)	Simplify as A as	0
			a <sup>9</sup>
			(Total for Question is 3 marks)
2.	(a)	Expand $2m(m+3)$	
			2m <sup>2</sup> +6m
	(b)	Factorise fully $3xy^2 - 6xy$	(1)
			3xy(y-2)
			(2) (Total for Question is 3 marks)
\3.	(a)	Expand $3(x + 4)$	
			3x+12
	/b)	Evened (v/v2 + 2)	(1)
	(D)	Expand $x(x^2 + 2)$	
			x <sup>3</sup> +2x <b>(2)</b>
	(c)	Factorise $x^2 - 6x$	, ,
			x(x-6)
			(1) (Total for Question is 4 marks)

				8x+29
(b) Fa	actorise	completely	3a²b + 6ab²	(2)
				3ab x (a+2b)
				(2) (Total for Question is 4 marks)
5.	(a)	Expand	3(2y-5)	
				6y-15 <b>(1</b> )
	(b)	Factorise cor	mpletely $8x^2 + 4$	xy
				4x x (2x+y)
				(2) (Total for Question is 3 marks)
6.	(a)	Factorise 3	x + 6	
				3(x+2) <b>(1</b> )
	(b)	Expand and	simplify $5(y-2)+2$	
				70, 16
				7y-16(2) (Total for Question is 3 marks)
7.	(a)	Factorise	4 <i>x</i> + 10 <i>y</i>	(Total for Quodion to o marks)
				2(2x+5y) <b>(1</b> )
	(b)	Factorise	$x^2 + 7x$	(1)
				x(x+7)(1)
				(1) (Total for Question is 2 marks)

Expand and simplify 5(x+7) + 3(x-2)

**4.** (a) 5x+35+3x-6

# **Solving Equations**

### Things to remember:

- "Solve" means to find the value of the variable (what number the letter represents).
- The inverse of + is and the inverse of x is ÷
- Work one step at a time, keeping you = signs in line on each new row of working.

### **Questions:**

1. Solve 4x + 3 = 194x=16

**2.** (a) Solve 6x - 7 = 38

6x = 45

*x* = ...7.5.....(2)

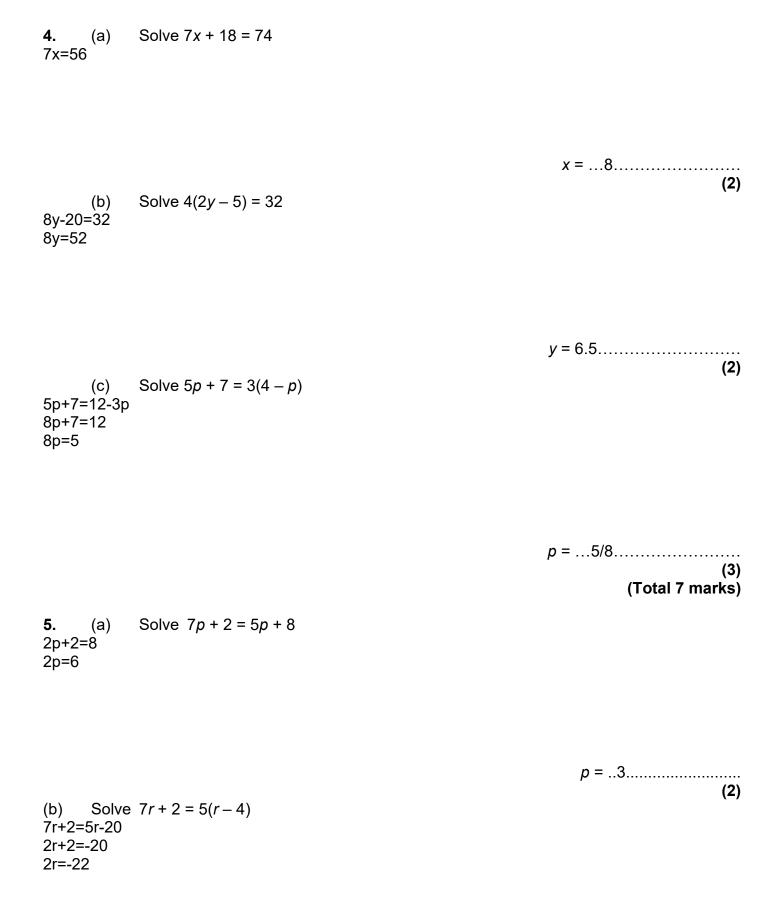
(b) Solve 4(5y - 2) = 40

20y-8=40 20y=48

> y = ....2.4.....(3) (Total 5 marks)

3. Solve 5(2y + 3) = 2010y+15=20 10y=5

y = ....0.5....(Total 3 marks)



*r* = -11.....

(Total 4 marks)

6. Solve 
$$4y + 1 = 2y + 8$$
  $2y+1=8$   $2y=7$ 

7. Solve 
$$4y + 3 = 2y + 8$$
  
 $2y+3=8$   
 $2y=5$ 

# **Rearranging Formulae**

### Things to remember:

- Firstly decide what needs to be on its own.
- Secondly move all terms that contain that letter to one side. Remember to move all terms if it appears in more than one.
- Thirdly separate out the required letter on its own.

### **Questions:**

**1.** Make *u* the subject of the formula

$$D = ut + kt2$$

d-kt2=ut

$$\frac{d-kt2}{t} = u$$

 $u = \dots \frac{d - kt2}{t}.$  (Total 2 marks)

**2.** (a) Solve 4(x+3) = 6

4x+12=6 4x=-6

...\_ -6 -

 $x = ... \frac{-6}{4} = \frac{-3}{2}...$  (3)

(b) Make t the subject of the formula v = u + 5t

v-u=5t

$$\frac{v-u}{5} = t$$

(Total 5 marks)

3. (a) Expand and simplify  $(x - y)^2$ 

..... $x^2 - 2xy + y^2$ .....(2)

(b) Rearrange a(q - c) = d to make q the subject.

Aq-ac=d Aq=d+ac

$$\frac{d+ac}{a}=q$$

Q = ...

4. Make x the subject of 
$$5(x-3) = y(4-3x)$$

$$x = \frac{4y+15}{5+3y}$$
....(Total 4 marks)

$$P = \frac{n^2 + a}{n + a}$$

Rearrange the formula to make a the subject.

$$P(n+a)=n^2+a$$
  
 $Pn+pa=n^2+a$   
 $Pn-n^2=a-pa$   
 $Pn-n^2=a(p-1)$ 

$$A = \dots \frac{pn-n^2}{p-1}$$
....(Total 4 marks)

$$\frac{x}{x+c} = \frac{p}{q}$$

Make *x* the subject of the formula.

$$X = \dots \frac{pc}{q-p}$$
.....(Total 4 marks)

## **Linear Simultaneous Equations**

### Things to remember:

- 1. Scale up (if necessary)
- 2. Add or subtract (to eliminate)
- 3. Solve (to find x)
- 4. Substitute (to find y) (or the other way around)

### **Questions:**

**\*1.** The Singh family and the Peterson family go to the cinema.

The Singh family buy 2 adult tickets and 3 child tickets.

They pay £28.20 for the tickets.

The Peterson family buy 3 adult tickets and 5 child tickets.

They pay £44.75 for the tickets.

Find the cost of each adult ticket and each child ticket.

2a+3c=28.2 3a+5c=44.75 10a+15c=141 9a+15c=134.25 a=f6.75 13.5+3c=28.2 3c=14.7 C=f4.90

(Total for question = 5 marks)

2. Solve the simultaneous equations

$$3x + 4y = 5$$

$$2x - 3y = 9$$

$$9x+12y=15$$

$$8x-12y=36$$

8x-12y=36 17x=51 x=3

6-3y=9 -3y=3

y=-1

*x* = ...3.....

*y* = ...-1.....

(Total for Question is 4 marks)

3.	Solve the	e simultaneous	equations
<b>J</b> .	O O I V C LI I C	, sirriditaricous	cqualions

$$4x + 7y = 1$$
  
 $3x + 10y = 15$   
 $40x+70y=10$   
 $21x+70y=105$   
 $-19x=95$   
 $x=-5$   
 $-20+7y=1$   
 $7y=21$ 

y=3

(Total for Question is 4 marks	)
<i>y</i> =3	
<i>x</i> =5	•

4.

Solve 
$$2x + 3y = \frac{2}{3}$$

$$3x - 4y = 18$$

5.	Solve the simultaneous equations $4x + y = 25$ x - 3y = 16
12x+3y	=75
x-3y=16	5
13x=91	
x=7	
7-3y=16	5
-3y=9	
y=-3	

x =	7	
v =	3	
,	(Total for Question is 3	

Solve the simultaneous equations 3x - 2y = 77x + 2y = 13

7. A cinema sells adult tickets and child tickets.

The total cost of 3 adult tickets and 1 child ticket is £30

The total cost of 1 adult ticket and 3 child tickets is £22

Work out the cost of an adult ticket and the cost of a child ticket.

3a+1c=30 1a+3c=22 9a+3c=90 8a=68 a=£8.50 8.5+3c=22 3c=13.5 c=£4.50

adult ticket	£8.50
	child ticket £4.50(Total for question = 4 marks)

\*8. Paper clips are sold in small boxes and in large boxes.

There is a total of 1115 paper clips in 4 small boxes and 5 large boxes.

There is a total of 530 paper clips in 3 small boxes and 2 large boxes.

Work out the number of paper clips in each small box and in each large box.

4s+5l=1115 3s+2l=530 8s+10l=2230 15s+10l=2650 7s=420 small=60 180s+2l=530 2l=350 large=175

(Total for Question is 5 marks)

# **Expand and Factorise Quadratics**

### Things to remember:

- Use FOIL (first, outside, inside, last) or the grid method (for multiplication) to expand
- For any quadratic  $ax^2 + bx + c = 0$ , find a pair of numbers with a sum of b and a product of ac to factorise.

Λ.	100	tia	ns:
ωı	ıes	tio	ns:

Questions:	
<b>1.</b> Expand and simplify ( <i>r</i> .	(m+7)(m+3)
m <sup>2</sup> +7m+3m+21	
	$m^2$ +10m+21(Total for question = 2 marks
2. (a) Factorise 6 +	3(2+3x)
(b) Factorise y <sup>2</sup>	(1)
(c) Factorise 2 <i>p</i>	$(1)^2 - p - 10$
$p^2$ -p-20 (p+4)(p-5) (p+2)(2p+5)	
	(p+2)(2p+5
	(2) (Total for Question is 4 marks
3. Solve, by factorising, the	ne equation $8x^2 - 30x - 27 = 0$
x <sup>2</sup> -30x-216=0 (	
	(Total for Question is 3 marks)

(x+4)(x-1)	$(x + m)^2 + n$ where $r$	- 2 <i>x</i> – 8 in the form	Write	5.
(Total for question is 2 marks		pand 4(3 <i>x</i> + 5)	(a)	6.
	2(x-4) + 3(x+5)	pand and simplify	(b)	
5x+7(2)	(x+4)(x+6)	pand and simplify	(c)	
x²+10x+24(2 (Total for Question is 5 marks	+ 4	ctorise $x^2 + 5x - $	(a)	7.
(x+4)(x+1)(2x+5)	(3x-1)(2x+5)	pand and simplify	(b)	
6x²+13x-5(2 (2 (Total for Question is 4 marks				

Factorise  $x^2 + 3x - 4$ 

8.	(a)	Expand	3(2+t)		6+3t
	(b)	Expand	3x(2x+5	)	(1)6 <i>x</i> <sup>2</sup> +15x
	(c)	Expand	and simplify	v (m + 3)(m + 10)	(2)
					$m^2$ +12m+30(2) (Total for Question is 5 marks)
9.	(a)	Factoris	se	$x^2 + 7x$	
	(b)	Factoris	se	<i>y</i> <sup>2</sup> – 10 <i>y</i> + 16	x(x+7)(1)
	(t+1)	(i) F	actorise	2t <sup>2</sup> + 5t + 2	(y-8)(y-2)(2)
(t+2)	(2t+1)	T		whole number. on 2 <i>t</i> <sup>2</sup> + 5 <i>t</i> + 2 ca	(t+2)(2t+1) n never have a value that is a prime number.
					(3) (Total for Question is 6 marks)

# **Using the Quadratic Formula**

### Things to remember:

• For any quadratic,  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### **Questions:**

1. Solve  $3x^2 + 7x - 13 = 0$ Give your solutions correct to 2 decimal places.

2. Solve the equation

$$2x^2 + 6x - 95 = 0$$

Give your solutions correct to 3 significant figures.

3. Solve  $x^2 + 3x - 5 = 0$ Give your solutions correct to 4 significant figures.

1.193 or –4.193

Solve this quadratic equation.				
	$x^2 - 5x - 8 = 0$ e your answers correct to 3 significant figures.	Give		
(3 marks)	x =6.27or x =1.27			
	Solve $x^2 - 2x - 1 = 0$ Give your solutions correct to 2 decimal places.	(a)	5.	
(3)	0.41 or 2.41	(b)		
(3) (6 marks)	2.41 or –0.41			

Solve  $x^2 + x + 11 = 14$ 

6.

(a)

 $y = x^2 + x + 11$  The value of y is a prime number when x = 0, 1, 2 and 3 The following statement is not true. ' $y = x^2 + x + 11$  is always a prime number when x is an integer' (b) Show that the statement is not true.

(4 marks)

# **Completing the Square**

### Things to remember:

- To complete the square:
  - 1. Halve the coefficient of x.
  - 2. Put in brackets with the x and square the brackets.
  - 3. Subtract the half-coefficient squared.
  - 4. Don't forget the constant on the end!
  - 5. Simply.
- For  $(x p)^2 + q = 0$ , the turning point is (p, q).

### **Questions:**

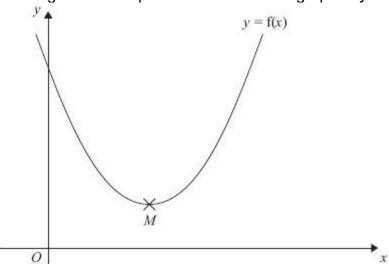
1. (i) Sketch the graph of  $f(x) = x^2 - 5x + 10$ , showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

(ii) Hence, or otherwise, determine whether f(x + 2) - 3 = 0 has any real roots. Give reasons for your answer.

(Total for question = 6 marks)

2.	(a)	Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$	where <i>a</i> , <i>b</i> , and <i>c</i> are integers.
			(3
	(b)	Hence, or otherwise, write down the coordinate of $y = 2x^2 + 16x + 35$	es of the turning point of the graph
			1) Total for question = 4 marks)
3.		expression $x^2 - 8x + 21$ can be written in the form	$(x - a)^2 + b$ for all values of x.
	(a)	Find the value of <i>a</i> and the value of <i>b</i> .	
			=
		D =	=(3

The equation of a curve is y = f(x) where  $f(x) = x^2 - 8x + 21$ The diagram shows part of a sketch of the graph of y = f(x).



The minimum point of the curve is *M*. (b) Write down the coordinates of *M*.

(1) (Total for Question is 4 marks)

# **Nonlinear Simultaneous Equations**

### Things to remember:

- 1. Substitute the linear equation into the nonlinear equation.
- 2. Rearrange so it equals 0.
- 3. Factorise and solve for the first variable (remember there will be two solutions).
- 4. Substitute the first solutions to solve for the second variable.
- 5. Express the solution as a pair of coordinate where the graphs intersect.

### **Questions:**

1. Solve the equations

$$x^2 + y^2 = 36$$
$$x = 2y + 6$$

$$(2y+6)^2 + y^2 = 36$$

$$=4y^2 + 36 + 24y + y^2 = 36$$

$$5y^2 + 24y = 0$$

y(5y+24)=0

y=-24/5

y=0

0+6=6

x=6

-48/5+6=-18/5

x = -18/5

.....y=-24/5 y=0 x=6 x=-18/5....... (Total for Question is 5 marks)

3. Solve the simultaneous equations

$$x^2 + y^2 = 25$$

$$y = 2x + 5$$

$$x^2 + (2x + 5)^2 = 25$$

$$x^2 + 4x^2 + 20x + 25 = 25$$

$$5x^2 + 20x = 0$$

$$5x(x+4)=0$$

$$x=-4$$

$$x = .-4$$
 . . . . . . . and  $y = . . . -3$  . . . . . . . or

$$x = ..0$$
 . . . . . . and  $y = ..5$  . . . . . . . . . (Total for Question is 6 marks)

4. Solve the simultaneous equations  $x^2 + y^2 = 9$ 

x + y = 2

Give your answers correct to 2 decimal places.

x=2-y  

$$(2-y)^2 + y^2 = 9$$
  
 $4 + y^2 - 4y + y^2 = 9$   
 $2y^2 - 4y - 5 = 0$ 

or  $x = \dots -0.87 \dots y = \dots 2.87 \dots$ 

(Total for Question is 6 marks)

5. Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$
  
 $y - 2x = 5$ 

Y = 2x + 5

$$x^{2} + (2x + 5)^{2} = 25$$
  

$$x^{2} + 4x^{2} + 20x + 25 = 25$$
  

$$5x^{2} + 20x = 0$$

5x(x+4)=0

x=-4

X=0

-8+5=-5

y=-3

0+5=5

y=5

(Total for Question is 5 marks)

# **Expanding more than two binomials**

### Things to remember:

- Start by expanding two pair of brackets using the grid or FOIL method.
- Then expand the third set of brackets.
- Use columns to keep x³, x² etc in line to help with addition.

### **Questions:**

1. Show that

$$(x-1)(x+2)(x-4) = x^3 - 3x^2 - 6x + 8$$

for all values of x.

$$(x^{2} - 5x + 4)(x + 2)$$

$$= x^{3} - 5x^{2} + 2x^{2} - 10x + 4x + 8$$

$$= x^{3} - 3x^{2} - 6x + 8$$

.....
$$x^3 - 3x^2 - 6x + 8$$
..... (Total for question is 3 marks)

2. Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$
 for all values of x.

$$(3x^{2} + 14x - 5)(4x - 3)$$

$$= 12x^{3} - 9x^{2} + 56x^{2} - 42x - 20x + 15$$

$$= 12x^{3} + 47x^{2} - 62x + 15$$

$$\dots 12x^3 + 47x^2 - 62x + 15\dots$$

(Total for question is 3 marks)

3. Show that

$$(x-3)(2x+1)(x+3) = 2x^3 + x^2 - 18x - 9$$
 for all values of x.

32

$$(2x^{2} - 5x - 3)(x + 3)$$

$$=2x^{3} - 5x^{2} - 3x + 6x^{2} - 15x - 9$$

$$=2x^{3} + x^{2} - 18x - 9$$

(Total for question is 3 marks)

 $(2x + 1)(x + 6)(x - 4) = 2x^3 + ax^2 + bx - 24$ 4. for all values of x, where a and b are integers. Calculate the values of a and b.

$$(x^{2} + 2x - 24)(2x + 1)$$

$$=2x^{3} + x^{2} + 4x^{2} + 2x - 48x - 24$$

$$=2x^{3} + 5x^{2} - 46x - 24$$